**1.**

**Barry and Garry decide to throw two fair dice. If the sum of outcomes of two dice is greater than or equal to 10, then Barry is declared winner on that throw. Otherwise, Garry is declared winner. After each throw, the winner receives 10$ from the looser. If Barry starts with 80$ and Garry with 20$, then what is the probability that Barry will wipe Garry out?**

**p = P{sum=10} + P{sum=11} + P{sum=12} = 3/36 + 2/36 + 1/36 = 6/36 = 1/6**

**q = 1 – p = 5/6**

**here, coin is equivalent to 10$, so Barry and Garry starts with 8 and 2 coins, respectively.**

**starting coins = i = 80/10 = 8**

**total coins in the system N = (80+20) / 10= 10**

**now put these values in the formula (see: Lec17-Markov Chains (Shuvo).pdf)**

**2.**

**Barry and Garry decide to flip two fair coins. If they get at least one head from a flip, then Barry is declared winner on that flip. Otherwise, Garry is declared winner. After each flip, the winner receives 20$ from the looser. If Barry starts with 40$ and Garry with 120$, then what is the probability that Barry will wipe Garry out?**

**p = P{1 head} + P{2 heads} = 2/4 + ¼ = ¾**

**q = 1 – p = ¼**

**starting coins = i = 40/20 = 2**

**N = (40+120) / 20= 8**

**now put these values in the formula (see: Lec17-Markov Chains (Shuvo).pdf)**

**3.**

**What do you understand by M/M/1, M/G/1, M/M/k and M/G/k queuing models?**

**The first ‘M’ denotes “Markovian” (i.e., events occur one by one, or the system follows discrete time model).**

**The second ‘M’ denotes ‘Memoryless’ … the arrival times follow memoryless distribution … For example, if the inter-arrival times follow exponential distribution, then they fulfill the memoryless property**

**The last ‘1’ indicates there is only ONE single server.**

**M/M/k: if multiple servers (i.e., *k* servers) are present in the system, then we have the M/M/k queueing model**

**M/G/1: … M: Markovian ….**

**G: any General distribution for the arrival times. For example, uniform or any other distribution for the inter-arrival times …**

**1: single server**

**M/G/k: M=Markovian model, G=General distribution for Arrivals, k: no of servers=*k***

**4.**

**Explain the following symbolic expressions in the context of the shoeshine shop discussed in the class.**

1. **P00 + P01:** The proportion of potential customers enters the system
2. **P10+P11+Pb1:** The proportion of potential customer that does not Enter the system
3. **P01+P10+2\*(P11+Pb1):** What is the mean number of customers in the system?

**d) … the expression of W …:** What is the average amount of time that an entering customer spends in the system?

**5. There are n people, numbered 1 to n, standing around a circle. From these people, every second remaining person is executed until only one person survives. Find the minimum value of n such that the person standing at the n/4-th position survives.**

**<small variation>**

**There are n people, numbered 1 to n, standing around a circle. From these people, every second remaining person is executed until only one person survives. Find the minimum value of n such that n >= 100 and the person standing at the n/4-th position survives.**

**<small variation>**

**There are n people, numbered 1 to n, standing around a circle. From these people, every second remaining person is executed until only one person survives. Find the minimum value of n such that n >= 100 and the person standing at the 25th position survives.**

**6.**

**Suppose we have a recurrence like :**

***f* ( *j* ) = α *j* , for 1 ≤ *j* < d … … … … (i)**

***f* ( *d* *n* + *j* ) = *c* *f* ( *n* ) + *β* *j* , for 0 ≤ *j* < *d* and *n* ≥ 1 … … … … (ii)**

**~~Solution of the above recurrence relation is given below:~~**

***~~f~~* ~~(~~ *~~b~~~~m~~**~~b~~~~m~~* ~~–1~~ ~~…~~ *~~b~~*~~1~~*~~b~~*~~0~~~~)~~*~~d~~* ~~= ( α~~*~~bm~~* ~~β~~*~~bm-~~*~~1~~ ~~β~~*~~bm-~~*~~2~~ ~~… β~~*~~b~~*~~1~~ ~~β~~*~~b~~*~~0~~ ~~)~~*~~c~~* ~~… … … … (iii)~~**

**Here, *d*= 4 , *c* = 5 , α *j* = 2*j* + 1 , *βj* = 3*j* – 1**

**Write the equations and compute *f* (50) using the radix based properties of a recurrence.**

**Solution:**

**The recurrences are:**

***f* (1) = 3**

***f* (2) = 5**

***f* (3) = 7**

***f* (4n) = 5 *f*(n) – 1**

***f* (4n+1) = 5 *f*(n) + 2**

***f* (4n+2) = 5 *f*(n) + 5**

***f* (4n+3) = 5 *f*(n) + 8**

**4 | 50**

**4 | 12 - 2**

**4 | 3 - 0**

**0 - 3**

***f* (50) = *f* (302)4 = ( α*3* β*0* β2 )*5* = (7 -1 5)5**

**= 7\*52 + (-1) \* 51 + 5 \* 50**

**= 175 - 5 + 5**

**= 175**

**To verify : f (50) = 5 \* f (12) + 5**

**= 5 \* { 5 f (3) – 1 } + 5**

**= 5 \* { 5\*7 – 1 } + 5**

**= 175 -5 + 5**

**= 175**

**7.**

**Write the following sum into its general form, then Convert the general sum into a recurrence. Solve the general recurrence to find the value of *Sn* for n = 100.**

*Sn* = 3 + 10 + 17 + … … … + 703

Sn can be generally written as : , with a=3 and b=7

Also find n, which is 100, because: a + nb = 703 🡺 3 + 7n = 703 🡺 **n = 100**

**Thus: we have to find S100** OR 

page 34-35 (scanned class lecture of Waishy)

Ans: 3\*101 + 100\*101/2 \* (7) = 353\*101

**8.**

**By using the Perturbation technique, Find a closed form expression for the Sum: **

**9.**

**At least 1 proof from Number Theory (May or May not from the Stern Brocot tree)**

**IF 2 proofs, then one must be from Stern Brocot tree!!!**

**­**

**10.**

**Find the value of . (Note that: The subscript *k* starts from 1, NOT from 0.)**

** = ?**

**=?**

**­­**

**Q: compute the value of the following series of binomial coefficients**

** r = 20, n = 40**

****

** m = 5, n = 60**

**ans : write the formula (pages 106) to find r =20, n = 40 and ans =  (ans) (or, you may use calculator to compute, but probably this may overflow !!!)**

**Q:**

** = use formula – 20C0 – 21C1 – 22C2**

**= above result – 1 – 21 – 22\*21/1\*2**

**Q: compute the value of the following series of binomial coefficients**



**ans : write the formula (pages 106) to find m =5, n = 60 and ans =  (ans)**

**Q:**

****

**solution =  – **

**= apply formula twice to get = **

****

**===============================================================**

**Q: Find the minimum three values of n for which J(n) = n/4**

**J(n)=n/4 🡺 2*l*+1 = ( 2m+*l* ) /4 🡺 7*l* = 2m– 4**

**🡺 *l* = 1/7 \*(2m– 4) 🡺 m = 1, 2, 3, 4, 5, 6, 7, 8, …**

**Take only those values of m for which *l* is integer 🡺**

**m = (2, 5, 8);**

***l* = (0, 4, 36)**

**n = 2m+*l* = (4, 36, 292)**

**survivor = J(n) = 2*l*+1 = 1, 9, 73**

**==========================================================**